Infinity Operads and Applications to Geometry
Contributed Talks

Sergei Burkin: *Operads, twisted arrow categories and homotopy coherent structures.*

Several known categories are used to encode homotopy coherent structures: the simplex category, Segal's category, Connes cyclic category, Moerdijk-Weiss dendroidal category (the backbone of many approaches to ∞-operads), and Hackney-Robertson-Yau categories. In this talk, based on my work, we will see that these categories are constructed from operads. The construction has several equivalent definitions, all related to classical constructions in category theory and operad theory. The corresponding categories are endowed with algebraic patterns of Chu and Haugseng, and thus encode homotopy coherent structures via Segal conditions. One of the ways to obtain these categories is to take ∞-localization of the category of elements of the nerve of an operad. That the construction is indeed an ∞-localization is essentially a theorem of Tashi Walde. This explains why the above-mentioned categories correctly encode homotopy coherent structures. In contrast, the further generalization of this construction to PROPs in general gives wrong categories, since these are not ∞-localizations. In the final part we will outline the generalization of this construction to ∞-operads and explain the connection to configuration spaces and representation stability.

João Fernandes: *Stable moduli spaces of odd-dimensional triads.*

The study of the homotopy type of moduli spaces of manifolds has seen significant progress in recent years. A key result in this area is the work of Galatius and Randall-Williams, who computed the homology of stable moduli spaces of even-dimensional manifolds. Their approach has led to several applications in the study of diffeomorphism groups in even dimensions. In this talk, I will describe an analogous result in the context of odd-dimensional manifold triads. I will begin by outlining the even-dimensional case and then present the odd-dimensional setting in analogy. I will conclude with an application, joint with Samuel Muñoz-Echániz, to computing the rational homotopy type of diffeomorphism groups of solid tori.

Tallak Manum: (∞, n)*-operads and the Gray-Boardman-Vogt tensor product.*

The theory of $(\infty, 1)$-operads gave us a homotopy-coherent way to study operads in spaces, with a major advantage being the Lurie–Boardman-Vogt tensor product, an intrinsically homotopy-coherent alternative to the classical version.

In this talk, based on my upcoming work, I'll outline how to extend this framework to (∞, n)-operads and introduce a Gray-Boardman-Vogt tensor product for them. I will also explain how to obtain a universal property for this tensor product.

The theory of (∞, n)-operads should be a useful tool in describing higher lax algebra and may even help simplify certain computations in the $(\infty, 1)$-setting.

Alice Rolf: *Endomorphisms and Automorphisms of the Framed Little Disk Operad.*

In a recent paper, Horel–Krannich–Kupers proved that all endomorphisms of the little disk operad are automorphisms. This prompts the question whether this is true for a more general class of operads, that is little disk operads with an action of a group. In this talk we are going to show some conditions of the group acting on the little disk operad in order for every endomorphism to be an automorphism. We will use methods from a recent paper by Krannich-Kupers to translate this question about the mapping space $\mathbb{E}_d$ to the mapping space of the classifying space of that group over $B\mathrm{Aut}(\mathbb{E}_d)$.

Chandan Singh: *Characterizing the Grothendieck-Teichmüller Group through Cyclic Operad.*

In 2008, Budney showed that the operad of framed little disks admits a cyclic structure. In this work, we show that this translates to a cyclic structure on a groupoid model for the framed little disks: the operad of parenthesized ribbon braids. We extend the known action of the Grothendieck-Teichmüller group **GT** on the operad of parenthesised ribbon braids to its cyclic version. This provides a new characterization of **GT** as an automorphism group of the prounipotent cyclic operad of parenthesized ribbons. As a consequence, we exploit this **GT** action to provide a simple proof of the formality of the cyclic framed little disk operad. Finally, this action extends to the category of parenthesized tangles, and verifies the conjecture of Kessel-Turaev about Galois actions on framed tangles. The work discussed here sets the foundation for exploring **GT** actions on a broader class of knotted objects such as B-tangles, virtual tangles, and welded tangles, which will be addressed in our future work. This is a joint work with Marcy Robertson.

Natalie Stewart: *Equivariant Dunn–Lurie additivity.*

In this talk, I'll lift Dunn–Lurie's additivity result to $\mathbb{E}_V$; specifically, there exists a "Boardman–Vogt" presentably symmetric monoidal structure on G-∞-operads, whose internal hom $\underline{\mathrm{Alg}}_{\mathcal{O}}(\mathcal{C})$ specializes to a "pointwise" G-symmetric monoidal structure on $\mathcal{O}$-algebras in any G-symmetric monoidal ∞-category $\mathcal{C}$, and I'll sketch an equivalence of G-operads $\mathbb{E}_V \otimes \mathbb{E}_V \simeq \mathbb{E}_{V \oplus W}$. This yields a natural equivalence of ∞-categories $\mathrm{Alg}_{\mathbb{E}_{V\oplus W}}(\mathcal{C}) \simeq \mathrm{Alg}_{\mathbb{E}_V} \underline{\mathrm{Alg}}_{\mathbb{E}_W}(\mathcal{C})$; corollaries include construction of a natural $\mathbb{E}_V$-algebra structure on real topological Hochschild homology of $\mathbb{E}_{V \oplus \sigma}$-algebras. Time permitting, I'll sketch how to prove additivity with equivariant tangential structure.